On the non-homogeneous cubic diophantine equation with four unknowns

$$x^2+y^2+4\left((2k^2-2k)^2z^2-4-w^2\right)=(2k^2-2k+1)xyz$$

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Abstract: The non-homogeneous cubic diophantine equation with four unknowns given by $x^2+y^2+4\left((2k^2-2k)^2z^2-4-w^2\right)=(2k^2-2k+1)xyz$ is analyzed for its non-zero distinct integer solutions through applying the linear transformtions and reducing it to pythagorean equation.

Keywords: Cubic equation with four unknowns,Non-homogeneous cubic, Integral solutions, Pythagorean equation.

Introduction:

The cubic diophantine equations are rich in variety and offer an unlimited field for research [1,2]. In particular refer [3-24] for a few problems on cubic equation with 3 and 4 unknowns. This paper concerns with an interesting non-homogeneous cubic diophantine equation with four unknowns given by $x^2+y^2+4\left((2k^2-2k)^2z^2-4-w^2\right)=(2k^2-2k+1)xyz$ for determining its infinitely many non-zero distinct integral solutions by reducing it to pythagorean equation.

Method of Analysis:

The non-homogeneous cubic equation with four unknowns under consideration is

$$x^2+y^2+4\left((2k^2-2k)^2z^2-4-w^2\right)=(2k^2-2k+1)xyz \tag{1}$$

Employing the linear transformations

$$x=2X+2(2k^2-2k+1)z \quad , \quad y=4 \tag{2}$$

in (1),it reduces to the equation

$$X^2=(2k-1)^2z^2+w^2 \tag{3}$$

which is solved through different ways and thus,inview of (2),one obtains different sets of solutions to (1).

Way:1

To start with,observe that (3) is in the form of the well-known pythagorean equation.Employing the most cited solutions of the pythagorean equation and performing a few calculations,the following two sets of solutions to (1) are obtained:

Set:1

$$x = 2\left((2k-1)^2 p^2 + q^2 + 2pq(2k^2 - 2k + 1)\right), y = 4$$
$$z = 2pq, w = (2k-1)^2 p^2 - q^2$$

Set:2

$$x = 2(2k-1)\left(2k^2 p^2 - (2k^2 - 4k + 2)q^2\right), y = 4$$
$$z = (2k-1)(p^2 - q^2), w = 2(2k-1)^2 pq$$

Way:2

(3) can be written as the system of double equations as below:

$$X + (2k-1)z = w^2,$$
$$X - (2k-1)z = 1$$

Solving the above system of equations and using (2),the corresponding solutions to (1) are given by

$$x = 8k^2\alpha((2k-1)\alpha + 1) + 2, y = 4$$
$$z = 2(2k-1)\alpha^2 + 2\alpha, w = 2(2k-1)\alpha + 1$$

Way:3

(3) can be written in the form of ratio as

$$\frac{X+w}{z} = \frac{(2k-1)^2 z}{X-w} = \frac{\alpha}{\beta}, \beta \neq 0$$

which is equivalent to the system of equations

$$\beta X + \beta w - \alpha z = 0$$
$$-\alpha X + \alpha w + (2k-1)^2 \beta z = 0$$

Applying the method of cross-multiplication,one has

$$X = \alpha^2 + (2k-1)^2\beta^2,$$
$$w = \alpha^2 - (2k-1)^2\beta^2, z = 2\alpha\beta \qquad (4)$$

In view of (2), one has

$$x = 2\alpha^2 + 2(2k-1)^2\beta^2 + 4(2k^2-2k+1)\alpha\beta, y = 4 \qquad (5)$$

Thus,(4) and (5) represent the integer solutions to (1).

Note:

(3) may also be written in the form of ratio as

$$\frac{X+w}{(2k-1)z} = \frac{(2k-1)z}{X-w} = \frac{\alpha}{\beta}, \beta \neq 0$$

For this choice,the corresponding integer solutions to (1) are as below:

$$x = 2(2k-1)(\beta^2+\alpha^2) + 4\alpha\beta(2k^2-2k+1), y = 4$$
$$w = (\alpha^2-\beta^2)(2k-1), z = 2\alpha\beta$$

Conclusion:

In this paper, an attempt has been made to obtain many non-zero distinct integral solutions to the non-homogeneous cubic equation with four unknowns given by $x^2 + y^2 + 4((2k^2-2k)^2 z^2 - 4 - w^2) = (2k^2-2k+1)xyz$. As cubic equations are rich in variety, the readers may search for obtaining integer solutions to other choices of cubic equations.

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