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A NEW APPROACH ON REGULAR FUZZY GRAPH

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Abstract

1.1 Graph

Graph theory is tremendously useful in modelling the essential features of systems with finite components. Wide applications of graphical models are in the field of railway network, telephone network, communication problems, traffic network etc. Graph theoretic models can sometimes provide a useful structure upon which analytic techniques can be used. A graph is also used to model a relationship between a given set of objects. Each object is represented by a vertex and the relationship between them is represented by an edge if the relationship is unordered and by means of a directed edge if the objects have an ordered relation between them. Relationship among the objects need not always be precisely defined criteria; when we think of an imprecise concept, the fuzziness arises.

1.2 Fuzzy graph

A mathematical frame work to explain the concept of uncertainty in real life through the publication of a seminal paper is introduced by Zadeh [1]. A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set. Rosenfeld [2] was introduced the fuzzy graph using fuzzy relation, represents the relationship between the objects by precisely indicating the level of the relationship between the objects of the given set. Also he coined many fuzzy analogous graph theoretic concepts like bridge, cut vertex and tree. Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision.

Introduction:

1.3 Literature survey

Fuzzy graph is growing fast and has numerous applications in various fields. Many authors are investigated on graph theory [3- 11]. Kaufmann [12] and Kim [13] have studied the fuzzy subsets, fuzzy matrices, fuzzy sets and system. Mordeson and Nair [14] studied the fuzzy graphs and fuzzy hypergraphs. Mordeson and Yao [15] studied the fuzzy cycles and fuzzy trees. Order and size are studied in fuzzy graphs by Nagoorgani and BasheerAhamed [16]. Nagoorgani and BasheerAhamed [17] defined the status in fuzzy graph. Nagoorgani and BasheerAhamed [18] have introduced the strong and weak domination concepts in fuzzy graphs. Nagoorgani and Chandrasekaran [19] explained many results of degree of vertex of a fuzzy graph. Nagoorgani and Chandrasekaran [20-22] obtained some new concepts on fuzzy graph.

Mordeson [23] examined the fuzzy line graphs. Mordeson and Peng [24] studied some of operations on fuzzy graphs. Mordeson and Nair [25-26] have studied the cycles, co-cycles of a fuzzy graph and fuzzy hypergraphs. Fuzzy cycles and fuzzy trees are studied by Mordeson and Yao [27]. Nagoorgani and Jahir Hussain [28-29] studied the connected, global domination of fuzzy graph and fuzzy independent dominating set. The effective Fuzzy Euler and Fuzzy Hamiltonian Graph are studied by Nagoorgani and Jahir Hussain [30]. Nagoorgani and Radha [31] introduced the lower and upper truncations on fuzzy graph. Nagoorgani and Radha [32] introduced the complement and conjunction of truncations on fuzzy graph. Some Sequences are studied in Fuzzy Graphs by Nagoorgani and Radha [33].

The regular fuzzy graphs, total degree and totally regular fuzzy graphs are introduced by Nagoorgani and Radha [34]. In this, the regular fuzzy graphs and totally regular fuzzy graphs are compared through various examples. Muhammad Akram and Wieslaw A. Dudek [35] introduced the concepts of regular and totally regular bipolar fuzzy graphs. The vertex regular fuzzy graph, total degree and totally vertex regular fuzzy graph are introduced by Kailash Kumar Kakkad and Sanjay Sharma [36]. The concepts of regular fuzzy graph, partially regular fuzzy graph and full regular fuzzy graph are introduced and some of their properties are studied by Nagoorgani and Radha [37]. Radhaand Vijaya [38] obtained the necessary and sufficient conditions for the Join of two totally regular fuzzy graphs to be totally regular under some restrictions.

1.4 Basic definitions

Definition 1.4.1:

By graph, we mean a pair G* = (V, E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*. We write $xy \in E$ to mean $\{x, y\} \in E,$ and if $e = xy \in E,$ we say x and y are adjacent.

Formally, given a graph G* = (V, E), two vertices $x, y \in V$ are said to be neighbors, or adjacent nodes, if $xy \in E.$

The neighborhood of a vertex v in a graph G* is the induced subgraph of G* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted N(v). The degree deg(v) of vertex v is the number of edges incident on v or equivalently, deg(v) = |N(v)|. The set of neighbors, called a (open) neighborhoodN(v) for a vertex v in a graph G*, consists of all vertices adjacent to v but not including v, that is, $N(v) = \{u \in V \mid vu \in E\}.$ When v is also included, it is called a closed neighborhoodN[v], that is, $N[v] = N(v) \bigcup \{v\}.$

A regular graph is a graph where each vertex has the same number of neighbors, i.e., all the vertices have the same open neighborhood degree.

A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge.

An isomorphism of graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v1 and v2 of G_1^* are adjacent in G_1^* if and only if f(v1) and f(v2) are adjacent in G_2^*. Isomorphic graphs are denoted by $G_1^* \cong G_2^*.$

In graph theory, the line graph L(G*) of a simple graph G* is another graph L(G*) that represents the adjacencies between edges of G*. Given a graph G*, its line graph L(G*) is a graph such that:

- each vertex of *L*(*G**) represents an edge of *G**; and
- two vertices of *L*(*G**) are adjacent if and only if their corresponding edges share a common endpoint (''are adjacent'') in *G**.

Let G* = (V, E) be an undirected graph, where $V = \{v_1, v_2, ..., v_n\}$. Let $S_i = \{v_i, x_{i1}, ..., x_{iq_i}\}$ where $x_{ij} \in E$ has vertex $v_i, i = 1, 2, ..., n, j = 1, 2, ..., q_i$. Let $S = \{S_1, S_2, ..., S_n\}$. Let $T = \{S_i S_j \mid S_i, S_j \in S, S_i \cap S_j \neq \phi, i \neq j\}$. Then P(S) = (S, T) is an intersection graph and P(S) = G*. The line graph L(G*) is by definition the intersection graph P(E). That is, L(G*) = (Z, W) where $Z = \{\{x\} \cup \{u_x, v_x\} \mid x \in E, u_x, v_x \in V, x = u_x v_x\}$ and $S_x = \{x\} \cup \{u_x, v_x\}, x \in E.$

Definition 1.4.1:

For a given graph G, the d2 – degree of a vertex v in G, denoted by d2(v) means number of vertices at a distance two away from v.

Definition 1.4.1:

A graph G is said to be (2, k)-regular (d2 –regular) if d2(v) = k, for all v in G. We observe that (2, k)-regular and semiregular graphs and d2-regular graphs are same.

Definition 1.4.1:

A graph G is said to be (r, 2, k) –regular if d(v) = r and d2(v) = k, for all v in G.

Definition 1.4.1:

A fuzzy subset of a non-empty set V is a mapping $\sigma : S \to [0,1]$ which assigns to each element 'x' in V a degree of membership, $0 \le \sigma(x) \le 1$.

Definition 1.4.1:

A fuzzy relation on *V* is a fuzzy subset of *V* × *V*. A fuzzy relation μ on *V* is a fuzzy relation on the fuzzy subset σ if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all *x*, *y* in *V* where $\wedge$ stands for minimum. A fuzzy relation on the fuzzy subset σ is reflexive if $\mu(x, x) = \sigma(x)$ for all $x \in V$. A fuzzy relation μ on *V* is said to be symmetric if $\mu(x, y) = \mu(y, x)$ for all x, $x, y \in V$. $\sigma^* = \sup(\sigma) = \{u \in V / \sigma(u) > 0\}$. $\mu^* = \sup(\mu) = \{(u,v) \in V \times V / \mu(u,v) > 0\}$.

Definition 1.4.1:

A fuzzy graph G is a pair of functions $G:(\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relations on σ. The underlying crisp graph of $G:(\sigma, \mu)$ is denoted by G*(V, E) where $E \subseteq V \times V$. A fuzzy graph G is complete if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edges between u and v.

(σ', μ') is a fuzzy sub graph or a partial fuzzy sub graph of (σ, μ) if $\sigma' \subseteq \sigma$ and $\mu' \subseteq \mu$; that is if $\sigma'(u) \le \sigma(u)$ for every $u \in V$ and $\mu'(e) \le \mu(e)$ for every $e \in E$.

(σ', μ') is a fuzzy spanning sub graph of (σ, μ) if $\sigma' \subseteq \sigma$ and $\mu' \subseteq \mu$; that is if $\sigma'(u) = \sigma(u)$ for every $u \in V$ and $\mu'(e) \le \mu(e)$ for every $e \in E$.

For any fuzzy subset n of V such that $v \subseteq \sigma$, the fuzzy sub graph of (σ, μ) induced by vis the maximal fuzzy sub graph of (σ, μ), that has fuzzy vertex set n and it is the fuzzy sub graph (v, τ) where $\tau(u, v) = v(u) \wedge v(v) \wedge \mu(u, v)$ for u, v in V.

Example 1.4.1:

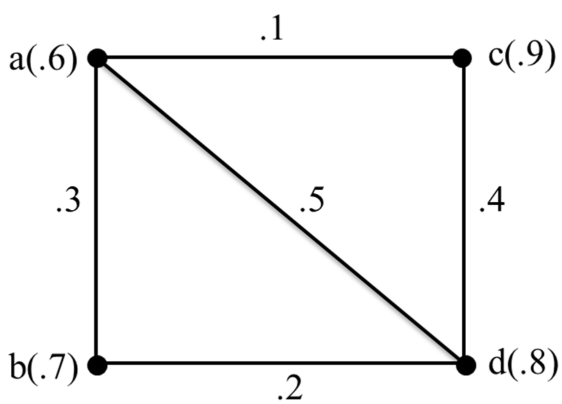

Fig. 1.1 $G:(\sigma, \mu)$

$G:(\sigma, \mu)$ is a fuzzy graph with the underlying set V = {a, b, c, d} where $\sigma: V \to [0, 1]$, $\mu: V \times V \to [0, 1]$, are defined as $\sigma(a) = .6,\ \sigma(b) = .7,\ \sigma(c) = .8,\ \sigma(d) = .9,\ \mu(a, b) = .3,$ $\mu(b, d) = .2,\ \mu(d, c) = .4,\ \mu(a, d) = .5.$

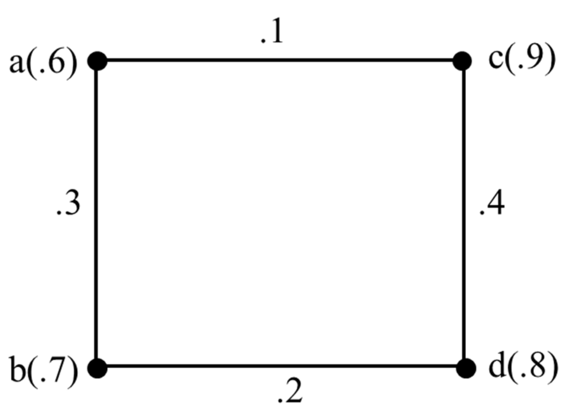

Fig. 1.2: $H:(\sigma', \mu')$ is a fuzzy sub graph of the fuzzy graph $G:(\sigma, \mu)$.

Definition 1.4.2:

Let $G:(\sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u)=\sum_{u\neq v}\mu(uv).$ Since $\mu(uv)>0$ for $uv\in E$ and $\mu(uv)=0$ for $uv\notin E,$ this is equivalent to $d_G(u)=\sum_{uv\in E}\mu(uv).$ The minimum degree of G is $\delta(G)=\wedge\{d(v)/v\in V\}.$

The maximum degree of G is $\Delta(G)=\vee\{d(v)/v\in V\}.$

The order of a fuzzy graph is $O(G)=\sum\sigma(u).$

Definition 1.4.2:

Let $G:(\sigma, \mu)$ be a fuzzy graph on G*:(V, E). The total degree of a vertex u is defined as $td(u)=\sum\mu(u, v)+\sigma(u)=d(u)+\sigma(u),\ uv\in E.$

Definition 1.4.2:

If each vertex of *G* has the same total degree *k*, then *G* is said to be totally regular fuzzy graph of degree *k* or *k*-totally regular fuzzy graph.

Definition 1.4.3:

The strength of connectedness between two vertices u and v is $\mu^{\infty}(u,v)=\sup\{\mu^{k}(u,v)/ k=1,2,.......\}$ where $\mu^{k}(u,v)=\sup\{\mu(uu_1)\wedge\mu(u_1u_2)\wedge......\wedge\mu(u_{k-1}v)/u_1,u_2,....,u_{k-1}\in V\}.$

Definition 1.4.4:

An edge *uv* is a fuzzy bridge of $G:(\sigma, \mu)$ if deletion of *uv* reduces the strength of connectedness between pair of vertices.

Definition 1.4.5:

A vertex *u* is a fuzzy cutvertex of $G:(\sigma, \mu)$ if deletion of *u* reduces the strength of connectedness between some other pair of vertices.

Definition 1.4.6:

Let $G:(\sigma, \mu)$ be a fuzzy graph such that G*:(V,E) is a cycle. Then G is a fuzzy cycle if and only if there does not exist a unique edge xy such that $\mu(xy)=\wedge\{\mu(uv)/(uv)>0\}.$

Definition 1.4.7:

The order of a fuzzy graph G is $O(G)=\sum_{u\in V}\sigma(u).$ The size of a fuzzy graph G is $S(G)=\sum_{uv\in E}\mu(uv).$

Definition 1.4.8:
Let $G:(\sigma,\mu)$ be a fuzzy graph. The complement of G is defined as $\bar{G}:(\sigma,\bar{\mu})$ where $\bar{\mu}(x,y)=\sigma(x)\wedge\sigma(y)-\mu(x,y)$ for all $x,y\in V$.

Definition 1.4.9
The $\mu-$complement of G is denoted as $G^{\mu}:(\sigma^{\mu},\mu^{\mu})$ where $\sigma^{\mu}=\sigma$ and $\mu^{\mu}(u,v)=0$ if $\mu(u,v)=0$ and $\mu^{\mu}(x,y)=\sigma(x)\wedge\sigma(y)-\mu(x,y)$ if $\mu(x,y)>0.$

Remark 1.4.1:
Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ denote two fuzzy graphs. Let $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ be underlying crisp graph such that $|V_i|=p_i,\ i=1,2.$ Also, $d_{G_i^*}(u_i)$ denote degree of u_i in G_i^*.

Definition 1.4.10:
An edge (x, y) in $\mu*$ is an effective edge if $\mu(x,y)=\sigma(x)\wedge\sigma(y).$ A fuzzy graph G is said to be a strong fuzzy graph if $\mu(x,y)=\sigma(x)\wedge\sigma(y).$ for all (x, y) in $\mu*$.

Definition 1.4.11:
A fuzzy graph G is said to be a complete fuzzy graph if $\mu(x,y)$ is $\sigma(x)\wedge\sigma(y)$ for all x, y in $\sigma*$.

Definition 1.4.12:
If $\mu(x,y)>0$ then x and y are called neighbours, x and y are said to lie on the edge $e = (x, y)$. A path ρ in a fuzzy graph $G:(\sigma,\mu)$ is a sequence of distinct nodes $v_0, v_1, v_2, \ldots, v_n$ such that $\mu(v_{i-1},v_i)>0,\ 1\le i\le n.$ Here 'n' is called the length of the path. The consecutive pairs (v_{i-1},v_i) are called arcs of the path.

Definition 1.4.13:

Let $G:(\sigma,\ \mu)$ be a fuzzy graph. The d_2 -degree of a vertex u in G is $d_2(u)=\sum\mu^2(u,\ v),$ where $\mu^2(u,\ v)=\sup\{\mu(u,\ u_1)\wedge\mu(u_1,\ v)\}.$ Also, $\mu(uv)=0,$ for uv not in E.

The minimum degree d_2-degree of G is $\delta_2(G)=\wedge\{d_2(v):v\in V\}.$

The maximum degree d_2-degree of G is $\Delta_2(G)=\vee\{d_2(v):v\in V\}.$

Example 1.4.2:

Consider G^*: $(V,\ E)$ where $V = \{u,\ v,\ w,\ x,\ y\}$ and $E = \{uv,\ vw,\ wx,\ xy,\ yu\}$. Define $G:(\sigma,\ \mu)$ by $\sigma(u)=.2,\ \sigma(v)=.6,\ \sigma(w)=.5,\ \sigma(x)=.4,\ \sigma(y)=.3$ and $\mu(uv)=1,\ \mu(vw)=3,$ $\mu(wx)=.3,\ \mu(xy)=.2,\ \mu(yu)=.2.$

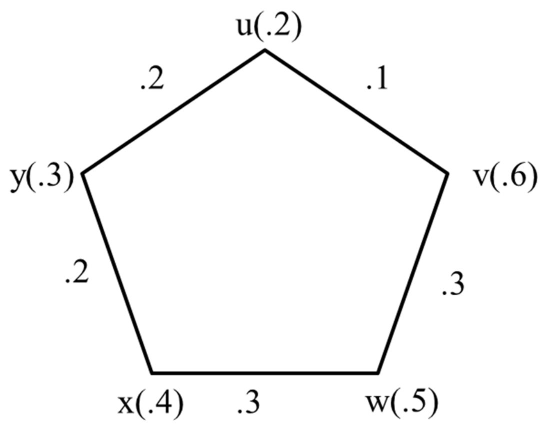

Fig. 1.4.3

Here, $d_2(u)$={.1∧.3}+{.2∧.2}=.1+.2=.3. $d_2(v)$={.1∧.2}+{.3∧.3}=.1+.3=.4.

$d_2(w)$={.3∧.2}+{.3∧.1}=.2+.1=.3. $d_2(x)$={.2∧.2}+{.3∧.3}=.2+.3=.5.

$d_2(y)$={.1∧.2}+{.2∧.3}=.1+.2=.3.

Definition 1.4.14:

Let $G:(\sigma,\ \mu)$ be a fuzzy graph on $G^*:(V,\ E)$. If $d_2(v) = k$ for all vV, then G is said to be $(2, k)$-regular fuzzy graph.

Example 1.4.5:

Consider G^*: $(V,\ E)$ where $V=\{u,\ v,\ w,\ x,\ y\}$ and $E=\{uv,\ vw,\ wx\}$. Define $G:(\sigma,\ \mu)$ by $\sigma(u)$ =.2, $\sigma(v)$ =.3, $\sigma(w)$= .4, $\sigma(x)$ = .5, and $\mu(uv)=.2,\ \mu(vw)=.2, \mu(wx)=.2.$

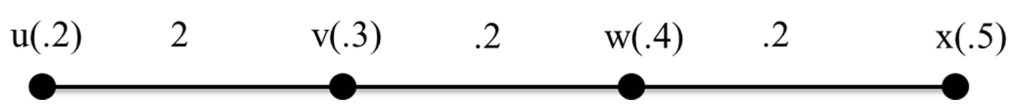

Fig. 1.4.5

Here $d_2(u)$ =.2, d_2(v) = .2, d_2(w) =.2, d_2(x) = .2. This graph is (2, .2) - regular fuzzy graph.

Example 1.4.6:
Any connected fuzzy graph with two vertices is (2, 0) - regular fuzzy graph.

Definition 1.4.15:

Let $G:(\sigma, \mu)$ be fuzzy graph on $G^*(V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \Sigma\, \mu^2(u,v) + \sigma(u) = d_2(u) + \sigma(u)$.

Example 1.4.7:
Let G: (σ, μ) be fuzzy graph on $G^*:(V, E)$, $V=\{u, v, w, x\}$ and $E=\{uv, vw, wx, xu\}$.

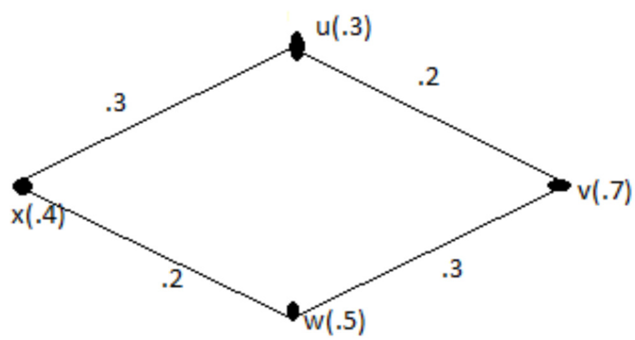

Fig. 1.4.7

Here $d_2(u)$ =.2, $d_2(v)$ = .2, $d_2(w)$ =.2, $d_2(x)$ =.2 and $td_2(u)$ =.5, $td_2(v)$ = .9, $td_2(w)$ =.7, $td_2(x)$ = .6. Each vertex has same d_2 - degree .2. So G is (2, .2) – regular fuzzy graph. But G is not totally (2, k)-regular fuzzy graph.

CONCLUSION:
Concluding remarks are presented below:
Chapter 1 dealt with the preliminaries of graph, fuzzy graph and their properties and reviewed the literature. In chapter 2, a characterization of regular fuzzy graphs on a cycle and properties of regular fuzzy graph are investigated. In this, the results are studied on k-regular fuzzy graph, r-totally regular fuzzy graph.

A characterization of (2, k) – regular fuzzy graphs on a path on four vertices, Barbell graph are presented in chapter 3. The chapter 4 deals with the vertex regular fuzzy graph and totally vertex regular fuzzy graph and properties of vertex regular fuzzy graphs. In chapter 5, edge regular bipolar fuzzy graph is studied.

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