RESEARCH ARTICLE OPEN ACCESS

Mean Labeling of Some Classes of Graphs

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Abstract:

The graph theory is one of the field of discrete mathematics which cuts across wide range of disciplines of human understanding not only in the areas of pure mathematics but also in variety of application areas ranging from computer science to social sciences and in engineering to mention a few. The later part of last century has witnessed intense activity in graph theory. Developments of computer science boost up the research work in the field. There are many interesting fields of research in graph theory. Some of them are domination in graphs, coloring in graphs, topological graph theory, fuzzy graph theory and labeling of discrete structures. Graph theory has been instrumental for analyzing and solving problems in areas as diverse as computer network design, urban planning, and molecular biology. Graph theory has been used to find the best way to route and schedule airplanes and invent a secret code that no one can crack. A branch of mathematics is called a Graph Theory, a graph bears no relation to the graphs that chart data, such as the progress of the stock market or the growing population of the planet.

Graph paper is particularly useful for drawing the graphs of Graph Theory. In Graph Theory, a graph is a collection of dots that may or may be connected to each other by lines. The principal object of the theory is a graph and its generalizations. The first problems in the theory of graphs were solutions of mathematical puzzles (the problem of the bridges of Konigsberg, the disposition of queens on a chessboard, transportation problems, the travelling-salesman problem, etc.). One of the first results in graph theory was a criterion on the possibility of traversing all edges of a graph without passing through any edge more than once; it was obtained by L. Euler in 1736 in solving the problem of the bridges of Konigsberg.

The **four-colour problem,** formulated in the mid-19th century, though a mere amusement puzzle at first sight, led to studies of graphs of both theoretical and applied interest. Certain studies in the mid-19th century contain results of importance to graph theory, obtained by solving practical problems. Thus, G. Kirchhoffs complete set of equations for currents and voltages in electric circuits

really amounts to representing the circuit by a graph with skeleton trees, with the aid of which linearly independent circuit systems are obtained. A. Cayley, starting from the problem of calculating the number of isomers of saturated hydrocarbons, arrived at the problems of listing and describing trees with certain properties, and solved some of them. In the 20th century, problems involving graphs began to arise not only in physics, chemistry, electrical engineering, biology, economics, sociology, etc., but also in mathematics itself in topology, algebra, probability theory, and number theory. At the beginning of the 20th century, graphs were used to represent certain mathematical objects and to formally state various discrete problems; besides the term "graph", other terms such as map, complex, diagram, network, labyrinth, were also employed.

Graph theory studies the properties of various graphs. Graphs can be used to model many situations in the real world, for example:

- The users of a social network and their friendships;
- The cities in a country and the streets that connect them;
- Telecommunication networks, like the Internet and the World Wide Web;
- Linguistic structure, for example syntactic structure of a sentence;

Recent years have seen an increased demand for the application of mathematics.

Graph theory has proven to be particularly useful to a large number of rather diverse fields. The exciting and rapidly growing area of graph theory is rich in theoretical results as well as applications to real-world problems. With the increasing importance of the computer, there has been a significant movement away from the traditional calculus courses and toward courses on discrete mathematics, including graph theory.

set topology. Mastoids are a certain abstraction of finite graphs. While a vivid area by itself they provide useful insights about some algorithmic aspects of graph theory.

Many real-life situation can be described by means of a diagram consisting of a set of points with lines joining certain pairs of points. Loosely speaking, such a diagram is what we mean by a graph. Graphs lend themselves naturally as models for a variety of situations. Instances of graphs abound: for example, the points might represent cities with lines representing direct flights between certain pairs of these cities in some airline system, or perhaps the lines represent pipelines between certain pairs of these cities in an oil network.On the other hand, the points might represent factories with lines representing communication links between them. Electrical networks, multiprocessor computers or switching circuits may clearly be represented by graphs.

Few applications of graph

Computer Networks: Graphs model intuitively model computer networks and the Internet. Often nodes will represent end-systems or routers, while edges represent connections between these systems.

Data Structures: Any data structure that makes use of pointers to link data together is making use of a graph of some kind. This includes tree structures and linked lists which are used all the time.

Molecules: Graphs can be used to model atoms and molecules for studying their interaction and structure among other things.

A lot of **matching problems** can be solved by graph. For example if you need to match processors to the work load or match workers to their jobs. A special graph, **trees,** has numerous applications in the computer science world. For example the syntax in programming language, or database indexing structure.

The Labeling of Graphs

The labeling of graphs is one of the potential areas of research due to its vital applications. The problems related to labeling of graphs challenges to our mind for their eventual solutions. This field has became a field of multifaceted applications ranging from neural network to bio-technology and to coding theory to computer science. Graph labeling were first introduced by A.Rosa during 1960. At present couple of dozens labeling techniques as well as enormous amount of literature is available in printed and electronic form on various graph labeling problems. The present work is aimed to discuss some graph labeling problems.

Introduction:

The present chapter is focused on mean labeling of some graphs. We investigate some new families of mean graphs and mean labeling of some special graph, namely a cubic graph on 8 vertices.

The concept of mean labeling was introduced by Somasundaram and Ponraj and they have proved that a path P_n, a cycle C_n are mean graphs for any $n \in N$ and $C_m \cup P_n$, $P_m \cup P_n$, $P_m \cup C_n$ are mean graph for any $m, n \in N$. They also proved that K_n, mean graph if and only if $n < 3$, $K_{1,n}$ is mean graph if and only if $n < 3$ and W_n is not a mean graph for $n > 3$ and $B_{m,n}$ is mean graph if and only if W_n is not a mean graph for $n > 3$.

In vaidya and Kanmani have proved that the graph obtained by the path union of k copies of cycle C_n is a mean graph, the graph obtained by joining two copies of cycle C_n by a path p_k is a mean graph.

We find the following observations while mean labeling of a graph

1. If $v \in V(G)$ and $d(v) \geq 2$ having label 0 then the edge label 1 can be produced only if v is adjacent to the vertex having label either 1 (or) 2.
2. Edge label 2 can be produced only if v is adjacent to the vertex with label either 3 (or) 4.
3. The edge label q can be produced only when the vertices with labels $q$and $q-1$.

Definition :4.1-Reqular graph

A Simple graph G is called a Regular graph if each vertex of G has an equal degree.

Definition 4.2- A cubic graph

A 3-regular simple graph is called a cubic graph

Illustration

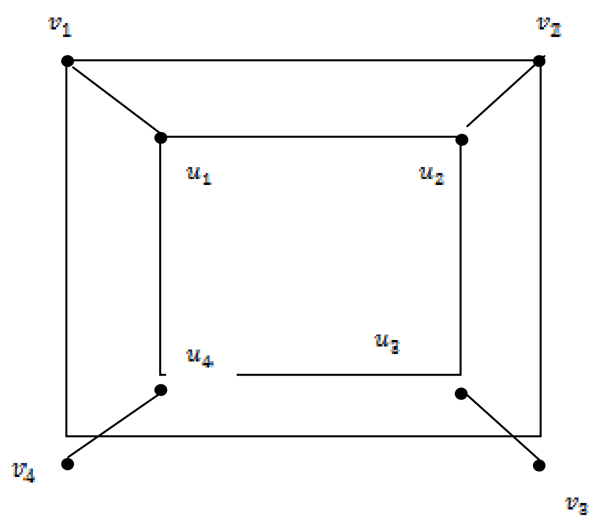

Figure :4.1- Cubic graph with 8 vertices

4.2 Mean Labeling

4.2.1 Mean Graph

A function f is called a mean labeling of graph G if f :V(G) → {0;1;2……..,q} is injective and the induced function f*: E(G) → {1,2, ………,q} defined as

$$f^*(e = uv) = \frac{f(u) + f(v)}{2}; \text{ if } f(u) + f(v) \text{ is even}$$

$$= \frac{f(u) + f(v) + 1}{2}; \text{ if } f(u) + f(v) \text{ is odd} \quad \text{is bijective.}$$

The graph which admits mean labeling is called a mean graph.

4.2.2 Illustration

In Figure 4.1 $K_{2,3}$ and its mean labeling is shown.

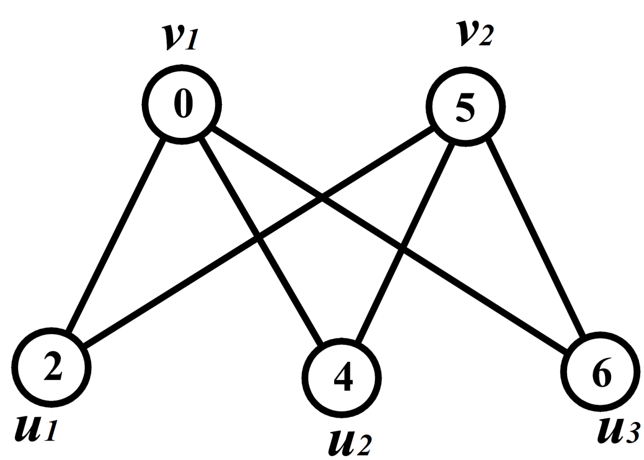

Figure 4.2 $K_{2,3}$ and its mean labeling

4.2.3 Some existing results

The concept of mean labeling was introduced by Somasundaram and Ponraj [41, 42, 43] and they have proved that

- P_n is mean graph for any $n \in N$.
- C_n is mean graph for any $n \in N$.

- $C_m \cup P_n$ is mean graph for any m, n $\in$ N.
- P_m X P_n is mean graph for any m, n $\in$ N.
- P_m X C_n is mean graph for any m, n $\in$ N.
- Kn is mean graph if an only if n<3.
- $K_{1,n}$ is mean graph if and only if n < 3
- $B_{m,n}$ is mean graph if and only if m<n+2

4.2.4 Square of a graph G

Square of a graph G denoted by G2 has the same vertex set as a G and two vertices are adjacent in G2 if they are at a distance 1 or 2 apart in G

4.2.5 Middle graph

The middle graph M(G) of a graph G is the graph whose vertex set is V(G)UE(G) and in which two vertices are adjacent if either they are adjacent edges of G or one is vertex of G and the other is an edgeincident on it .

4.2.6 Total graph

The total graph T(G) of a graph G is the graph whose vertex set v(G)UE(G) and two vertices are adjacent whenever they are either adjacent or incident in G

Theorem 4.1. $P_n^2$is a mean graph.

Proof. Let ν_1, ν_2, …………….νn be the vertices of path P_n. Define f: V(G) $\rightarrow$ {0, 1, 2, ……., q) as follows.

$f(v_i) = 2i-2$ for $i \leq (n - 1)$

$f(v_1) = 0$

$f(v_2) = 2$

$f(v_3) = 4$ and $f(v_n) = f\ 2(n-1)$

$f(v_i) = 2i – 3$ for $i = n$

Above defined labeling pattern provides mean labeling for P_n^2

That is, P_n^2 is a mean graph.

Illustration 4.1. The graph P_6^2 and the corresponding mean labeling is shown in Figure 4.2.

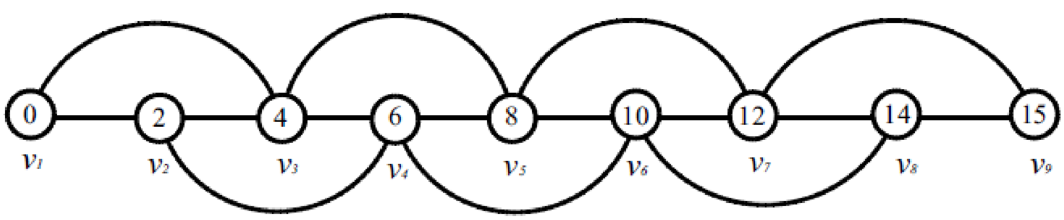

Figure 4.3 P_9^2 and the its mean labeling

Theorem 4.2 M(Pn) admits mean labeling.

Proof. Let $v_1,v_2,. . ……..$ vn be the vertices and $e_1,e_2,. . …. , e_{n-1}$ be the edges of path P_n. $M(P_n)$ consists of two types of vertices $\{v_1,v_2,. . . , v_n\}$ and $\{e_1,e_2,. . . , e_{n-1}\}$. Define $f :V(M(P_n))\rightarrow \{0, 1, 2,….q\}$ as follows.

$f(e_1) = 1$

$f(e_2) = 4$

$f(e_3) = 8$

$f(e_i) = 3i-1$; for $4 \leq i \leq (n-1)$

$f(v_1) = 0$

$f(v_i) = 2i-1$; for $2 \leq i \leq 4$

$f(v_i) = 3i-5$; for $5 \leq i \leq n$

The above defined function f provides mean labeling for $M(P_n)$. That is, $M(P_n)$ is a mean graph.

Illustration 4.2(a) The mean labeling for $M(P_6)$ is as shown in Figure 4.3

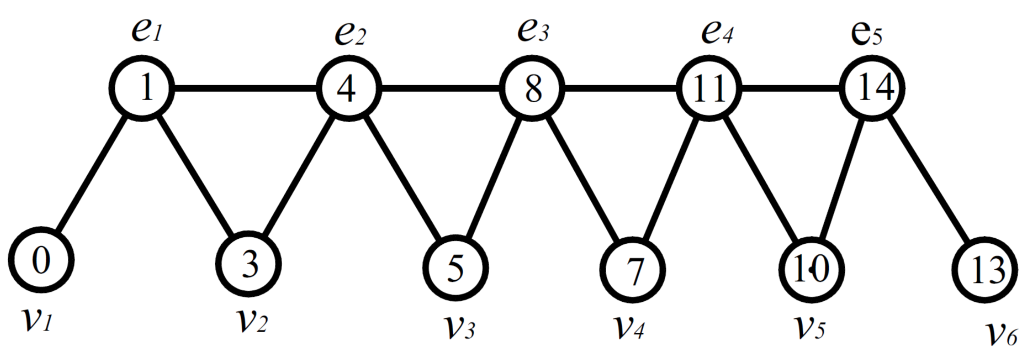

Figure: 4.4 $M(P_6)$ and its mean labeling

Illustration 4.2(b) The mean labeling for $M(P_7)$ is as shown in Figure 4.5

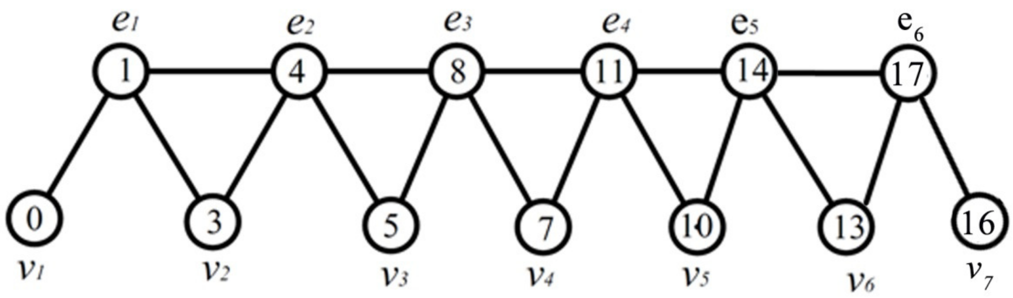

Figure: 4.5 $M(P_7)$ and its mean labeling

Theorem 4.3 Sunlet graphs are mean graphs.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of cycle C_n. Let G_1 be the new graph contains 2_n vertices $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ and 2_n edges (n edges o f cycle C_n together with n pendant edges. To define
$f : V(G_1) \rightarrow \{0,1,2,\ldots..q\}$ the following two cases are to be considered.
Illustration:
Case 1: n is odd.

$f(v_1) = 0$

$f(v_i) = 2i - 1$;for$2 \leq i \leq \frac{n+1}{2}$

$f(v_i) = 2i$;for$\frac{n+3}{2} \leq i \leq n$

$f(u_1) = 1$

$f(u_i) = 2i - 2$;for$2 \leq i \leq \frac{n+1}{2}$

$f(u_i) = 2i$;for$i = \frac{n+1}{2}$

$f(u_i) = 2i - 1$;for$\frac{n+3}{2} \leq i \leq n$

Case 2: n is even

$f(v_1) = 0$

$f(v_i) = 2i - 1$;for$2 \leq i \leq \frac{n}{2}$

$f(v_i) = 2i$;for$\frac{n+2}{2} \leq i \leq n$

And $f(u_1) = 1$

$f(u_i) = 2i - 2$;for$2 \leq i \leq \frac{n}{2}$

$f(u_i) = 2i$;for$\frac{n+2}{2} \leq i \leq n$

The above defined pattern covers all the possibilities and the graph under consideration admits mean labeling.

Illustration:4.3(a)

Case:1

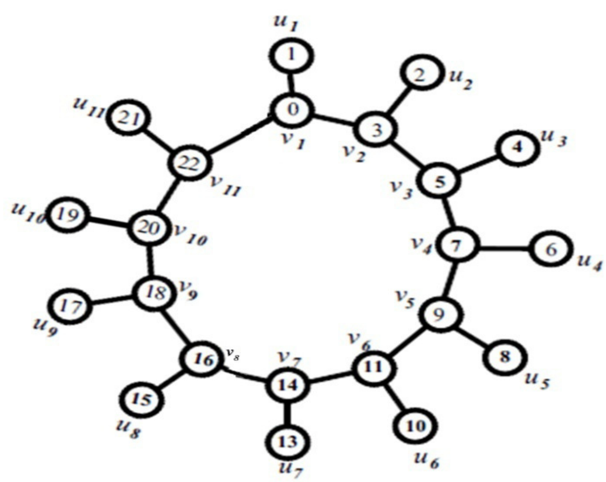

Figure 4.6 : The sunlet graph s_{11} is a mean graph.

Illustration 4.3(b)

Case: ii

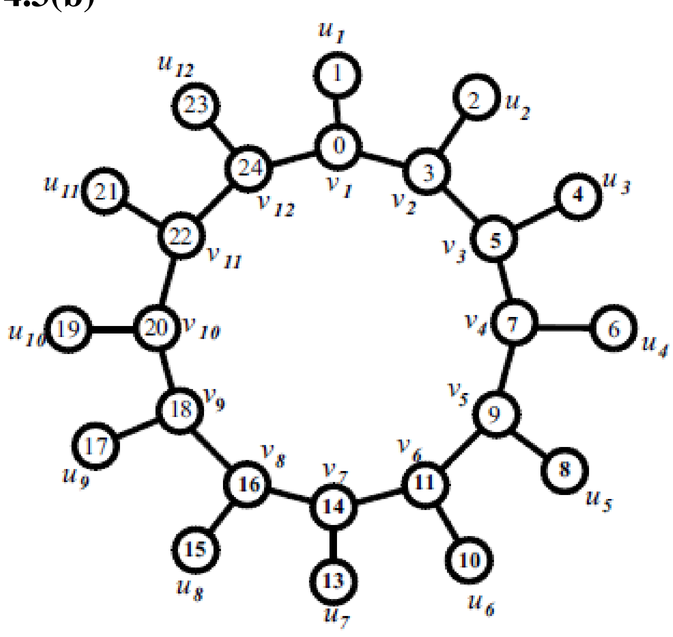

Figure 4.7: The sunlet graph s_{12} is a mean graph.

CONCLUSION:

This dissertation investigated mean labeling and Mean graphs. Further it examine vertex odd Mean and even Mean of some classes of graphs. It is intersecting to note that not all graphs are Mean graphs. It has been proved some new results for Mean labeling in the context of some graph operations on some standard graphs. To derive similar results in the context of different graph labeling problems and for various graph families is an open area of research.

Reference:

[1] J.A. Bondy and U.S.R Murthy, Graph Theory and applications (North-Holland), Newyork (1976).
[2] J.A. Gallian, A dynamic survey of labeling The Electronics journal of combinatories 17 (2014)
[3] F. Harary, Graph Theory, Addition Wesley, Reading, Massachusetts, 1972.
[4] Manickam. K and Marudai. M, odd mean labeling of graphs, Bulletin of pure and Applied Sciences Vol.25 E, No.1(2016), 149-153.
[5] Somasundaram.S and Ponraj.R, Mean labeling of Graphs, Nate. Acad, Sci. let. 26 (2003), 210-213.