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Regular Storngly Connected Sets in Topology

[1]A. Rathika , [2]R. Vaithiyalingam
[1]M.Phil Research Scholar,[2]Asst.Professor in Maths
Department of Maths, Prist University , Puducherry, India

Abstract:

Topology is an important branch of mathematics. Topological spaces show up naturally in almost every branch of mathematics. This has made topology one of the great unifying ideas of mathematics.

Topology can be formally defined as "the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map), especially those properties that are invariant under a certain kind of invertible transformation(called homeomorphism)'.

Topology is also used to refer to a structure imposed upon a set X, a structure that essentially 'characterizes' the set X as a topological space by taking proper care of properties such as connected, connectedness and compactness, upon transformation.

Introduction

REGULAR STRONGLY CONNECTED SETS IN TOPOLOGY

2.1 Introduction

A subset A of a topological space (X, τ) said to be regular open, if A=$Int(Cl(A))$ and A is called regular closed, if and only if, it is complement is regular open. The set of all regular open sets in X is denoted by R.O(X), [16], A subset A of a topological space X will be termed. Strongly connected if A⊂ U or A⊂ V whenever A⊂U ∪ V, V and U being open set in X [39], A subset A of X will be termed weakly disconnected. If it is not strongly connected [39].

A topological space X is totally weakly disconnected. If singleton sets are the only nonempty strongly connected sets [39]. In this work define a regular strongly connected sets and a regular weakly disconnected sets and investigate a relationship between these sets, also we shall define a regular totally weakly disconnected topological space and to introduce a relation between this space and a regular $\tau_1$as a separation axiom in topological space.

2.2 Regular strongly connectivity

Definition 2.1

A space X is connected if it cannot be decomposed as the union of two disjoint nonempty open sets.

Definition 2.2

A subset A of a topological space (X, τ) is said to be regular strongly connected, if and only if A⊂ U or A⊂ V whenever A⊂ U ∪ V, U and V being regular open sets in X.

Theorem 2.3

Every regular strongly connected sets is strongly connected sets.

Proof .

Let A is regular strongly connected set, then A⊂ U ∪ V, whenever A⊂ U or A⊂ V and U and V being regular open sets in X, since every regular open set is open set U and V are open set in X and A⊂ U ∪ V, hence A is strongly connected set.

Theorem 2.4

If A is strongly connected, then A is connected.

Proof. If A is disconnected.

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Then there exist open sets U and V for which A = (A∩U) ∪ (A∩V). A∩U $\neq\phi$, A∩V$\neq\phi$ and A∩U∩V $=\phi$ then A⊂U∪V, but A⊊U and A⊊V which contradicts A being strongly connected.

Theorem 2.5

If A is regular strongly connected, then A is connected.

Proof.

Easy by using definition (2.2) and theorem (2.3).

Remark 2.6

The converse of theorem (2.4), is not true. As shown by following example.

Example 2.7

Let X={1, 2, 3}, τ ={φ, X, {2}, {1, 2}, {2, 3}} it is clear that X is connected but not regular strongly connected.

Definition 2.8

A subset A of a topological space X is said to be regular weakly disconnected. if and only if it is not regular strongly connected.

Remark 2.9

If A is regular disconnected, then A is regular weakly disconnected.

11.Proof.

Since A is regular disconnected, then A is not regular connected and by theorem (2.4), A is not regular strongly connected and hence A is regular weakly disconnected.

Theorem 2.10

A subset A is regular weakly disconnected. If and only if, there exist two nonempty disjoint sets M and N each regular closed in A.

Proof.

Let A is regular weakly disconnected, then A is not regular strongly connected, which is mean there is no regular open set U and V.

Whenever A⊂U or A⊂V and A⊂U∪V so let M=U^c and N=V^cM and N are regular closed. Since A⊂U→ $A \subsetneq U^c = M$ or A⊂V→ $A \subsetneq V^c = N$ and A ⊊ M∪V, since A is not regular strongly connected this implies that M and N are two nonempty disjoint regular closed in A. The other hand is easy from a definition of regular closed sets in A.

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Corollary 2.11

A subset A is regular weakly disconnected. If and only if, there is exist two points x and y in A such that A∩ $Cl(x) \cap Cl(y) = \phi$, where $Cl(x)$ and $Cl(y)$ be the closure of x and y respectively.

Theorem 2.12

A topological space (X, τ) is regular strongly connected, if the only non-empty subset of X which is both regular open and regular closed in X is X itself.

Theorem 2.13

Suppose (X, τ) is regular strongly connected and let F be a regular closed subset of X then F is regular strongly connected.

Proof.

If F is not regular strongly connected. Then, f is regular weakly disconnected, then there exist regular open sets v and U for which F= (F∩ U)∪(F∩U). F∩U $\neq \phi$, F∩V $\neq \phi$ and F⊂U∪V $= \phi$ then F⊂U∪V, but F⊊U and F⊊V which make F is regular open, which contradicts with F as a regular closed.

Corollary 2.14

A space (X, τ) is regular strongly connected. If and only if every regular closed subset F is regular strongly connected.

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Proof

Suppose(X, τ) is regular strongly connected then by theorem (2.13). Every regular closed subset is connected.

Suppose that F is regular closed subset and F is regular strongly connected then F⊂U or F⊂U or F⊂V when ever F⊂U∪V, U and V being regular open sets in X, but F⊂U∪V and F is regular closed. Hence the only non empty subset of X is X itself which is mean that (X, τ) is regular strongly connected.

Definition 2.15

Let f be a mapping from topological space (X, τ) in to topological space (Y, F), then f is said to be continuous function if the inverse image of any open (closed) in Y is open (closed) in X.

Definition 2.16

Let f be a mapping from topological space (X, τ) in to topological space (Y, F), then f is said to be regular continuous if the inverse image of any open(closed) in Y is regular open(regular closed) in X.

Theorem 2.17

If f : (X, τ) → (Y, F) is regular continuous if A is regular strongly connected in X, then $f(A)$ is regular strongly connected in Y.

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Proof.

Let $f: X \to Y$ be a regular continuous and A is regular strongly connected in X. To prove that $f(A)$ is regular strongly connected in Y. Assume that $f(A)$ is not regular strongly connected in Y→ there exist U, V both regular open in Y

Such that $U \cap f[A] \neq \phi, V \cap f[A] \neq \phi$ and $(U \cap f[A]) \cap (V \cap f[A]) = \phi$ and

$(U \cap f[A]) \cup (V \cap f[A]) = f[A]$,

$f^{-1}(\phi) = f^{-1}(U \cap f[A]) \cap (V \cap f[A]))$

$= f^{-1}(U \cap V) \cap f[A])$

$= f^{-1}(U \cap V) \cap f^{-1}(f[A])$

$= f^{-1}(U) \cap f^{-1}(V) \cap A$

$= f^{-1}(U) f^{-1}(V)$

$A = f^{-1}(f[A])$

$= f^{-1}(U \cap f[A]) \cup (V \cap f[A])$

$= f^{-1}((U \cup V) \cap f[A])$

$= f^{-1}(U) \cup f^{-1}(V) \cap f^{-1}(f[A])$

$= f^{-1}(U) f^{-1}(V) \cap A$

$= f^{-1}(U) \cup f^{-1}(V)$

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Since f is regular strongly connected and U,V are regular open in Y impels $f^{-1}(U), f^{-1}(V)$ are regular open in X so X is not regular strongly connected which is contraction so $f[A]$ must be regular strongly connected in Y.

Definition 2.18

A topological space X is totally weakly disconnected, if singleton sets are the only non empty regular strongly connected sets.

Theorem 2.19

A topological space X is Rτ_1, if and only if, then regular totally weakly disconnected. Where R$\tau_1$be the regular τ_1-space.

Proof. **Necessity,**

Singleton sets are clearly regular strongly connected. Now suppose A is a subset of X with two or more points, let $x \neq y$ in A, then x and y are non empty disjoint regular closed subset of A and by theorem (2.17).

Sufficiency,

Let $x \in X$ and suppose $[x] \neq Cl[x], let\ y \in Cl[x] - [x]$ and let A=$[x, y]$, then every regular open set which contains also contains x and thus A is regular strongly connected which is contradiction.

Corollary 2.20

Let a topological space (X, τ) is regular totally weakly disconnected then X is RTO.

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Proof. Easy by using the fast that every Rτ_1- space is Rτ_2 - space and theorem (2.19).

Theorem 2.21

Every discrete space (X, D) defined by a regular open set is regular totally weakly disconnected.

Proof.

Let (X, D) be a regular discrete space and let $x \neq y$ in X, let $A = \{x\}, B = X - \{x\}$ are both non empty regular open disjoint sets whose union is X. Such that $x \in A$ and $y \in B \rightarrow (X, D)$ is regular totally weakly disconnected.

2.3 Regular Strong Local Connectivity

Definition 2.22

A topological space (X, τ) is said to be regular strongly locally connected at x, if for $x \in O \in \tau$, there exists a regular strongly connected open set G such that

$x \in$ G⊂O.

Remark 2.23

A topological spaces (x, τ) is regular strongly locally connected at each point of X.

General topology, also called point – set topology, established the foundational aspects of topology and investigates properties of topological spaces and concepts inherent to topological spaces.

The fundamental concepts in point – set topology are continuity, compactness and connectedness. Intuitively, continuous functions take nearby points to nearby points.

Compact sets are those that can be covered by finitely many sets of arbitrarily small size. Connected sets are sets that cannot be divided into two pieces that are far apart.

The words nearby, arbitrarily small and far apart can all be made precise by using open sets. If we change the definition of open set, we change what continuous functions, compact sets and connected sets are. Each choice of definition for open set is called a topology. A set with a topology is called a topological space.

Metric spaces are an important class of topological spaces where distance can be assigned a number called a metric. Having a metric simplifies many proofs and many of the most common topological spaces are metric spaces.

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